Variational Quantum Algorithm for Quantum Sensor Evaluation

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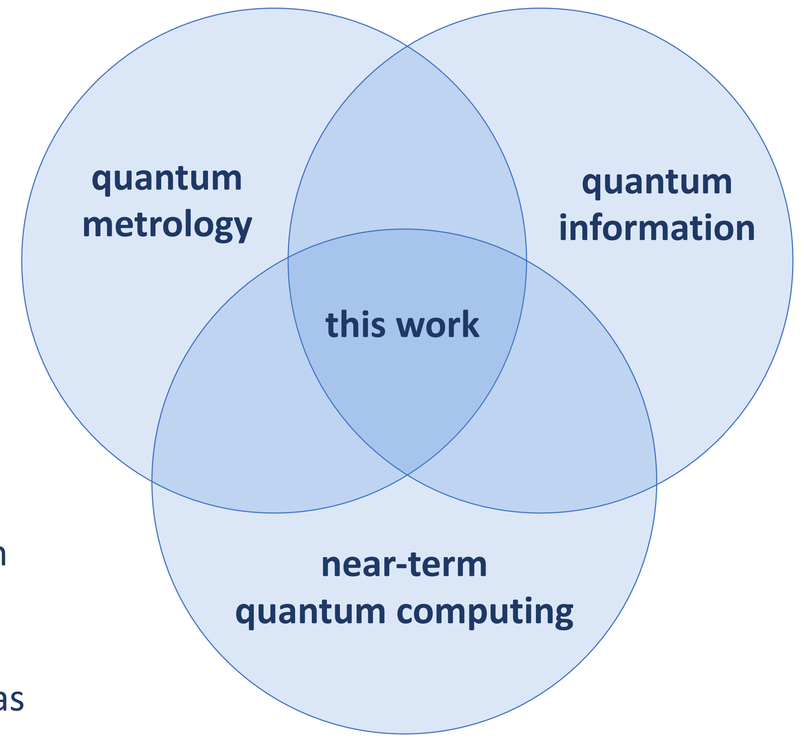
Introduction

Using quantum resources to enhance sensing capabilities has led to profound results in many fields (e.g., gravitational wave detection, biosensing, etc.)

The **quantum Fisher information (QFI)** is a fundamentally important quantity in quantum sensing because it quantifies the ultimate precision achievable in estimating an unknown parameter from a quantum state.



Problems:

1. computing the QFI for mixed states is computationally difficult
2. proposed methods of estimating the QFI have scaling that precludes their implementation on near-term quantum devices
3. most known methods require detailed knowledge of how the unknown parameter was encoded in the state (sensor dynamics)

We address these problems by introducing a new variational quantum algorithm called the **Variational Quantum Fisher Information Estimation (VQFIE) algorithm**.

Using novel, dynamics-agnostic bounds, VQFIE outputs a range in which the actual QFI lies. Additionally, VQFIE can be easily extended to **prepare the state that maximizes the QFI** for the application of quantum sensing.

Theoretical Background

Common scenario: Estimate an unknown parameter θ which parameterizes the state ρ_θ
Natural question: How well can one estimate this parameter?

Quantum Cramer-Rao Bound

$$\mathrm{Var}[\theta] \geq \frac{1}{m I(\theta;\rho_\theta)}$$

quantum Fisher information (QFI)

QFI is related to the standard fidelity between the **exact state,** ρ_θ , and the **error state,** $\rho_{\theta+\delta}$

exact state

$$\rho_\theta = e^{-i\theta G}\rho e^{+i\theta G}$$

error state

$$\rho_{\theta+\delta} = e^{-i(\theta+\delta)G}\rho e^{+i(\theta+\delta)G}$$

fidelity

$$F(\rho_\theta,\rho_{\theta+\delta}) = \mathrm{tr}\left[\sqrt{\sqrt{\rho_\theta}\rho_{\theta+\delta}\sqrt{\rho_\theta}}\right]$$

$$I(\theta;\rho_\theta) = \lim_{\delta\to 0} 8\left(\frac{1-F(\rho_\theta,\rho_{\theta+\delta})}{\delta^2}\right) = -4\lim_{\delta\to 0}\partial_\delta^2 F(\rho_\theta,\rho_{\theta+\delta})$$

limiting case intuition:
identical exact & error states minimize QFI
orthogonal exact & error states maximize QFI

geometric intuition:
the more peaked the fidelity between the exact & error states is, the higher the QFI

Truncated QFI Bound

1. **Compute m largest eigenvalues and associated eigenvectors** $\{\lambda_k, |\lambda_k\rangle\}_{k=1}^m$
2. **Order the eigensystem from largest to smallest** $\lambda_k \geq \lambda_{k+1}$
3. **Form exact state projectors** $\Pi_{\rho_\theta}^m = \sum_{k=1}^m |\lambda_k\rangle\langle\lambda_k|$
4. **Form exact and error truncated states**

exact truncated state

$$\rho_\theta^{(m)} = \Pi_{\rho_\theta}^m \rho_\theta \Pi_{\rho_\theta}^m = \sum_{k=1}^m \lambda_k |\lambda_k\rangle\langle\lambda_k|$$

error truncated state

$$\rho_{\theta+\delta}^{(m)} = \Pi_{\rho_\theta}^m \rho_{\theta+\delta} \Pi_{\rho_\theta}^m$$

We obtain the TQFI by replacing the **fidelity** by the **truncated generalized fidelity**[1]

$$\mathcal{I}_*(\theta;\rho_\theta^{(m)}) = 8\lim_{\delta\to 0}\frac{1-F_*(\rho_\theta^{(m)},\rho_{\theta+\delta}^{(m)})}{\delta^2}$$

where

$$F_*(\tau,\sigma) = F(\tau,\sigma) + \sqrt{(1-\mathrm{tr}[\tau])(1-\mathrm{tr}[\sigma])}$$

The generalized fidelity **upper bounds the fidelity,** and thus **lower bounds the QFI**

$$F(\rho_\theta^{(m)},\rho_{\theta+\delta}^{(m)}) \leq F_*(\rho_\theta^{(m)},\rho_{\theta+\delta}^{(m)}) \longrightarrow \mathcal{I}_*(\theta;\rho_\theta^{(m)}) \leq I(\theta;\rho_\theta)$$

Sub-QFI Bound

We also explored the use of an efficiently computable lower bound on the QFI we call the **sub-QFI**

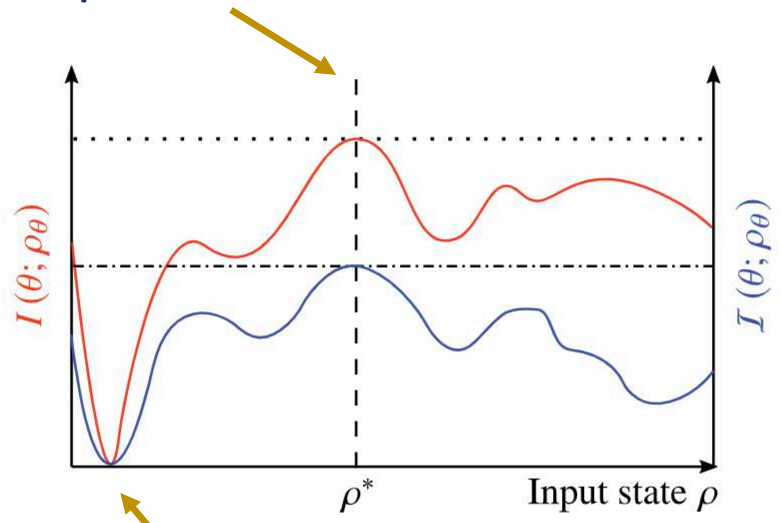
$$G(\rho_\theta,\rho_{\theta+\delta}) = \mathrm{tr}[\rho_\theta\rho_{\theta+\delta}] + \sqrt{(1-\mathrm{tr}[\rho_\theta^2])(1-\mathrm{tr}[\rho_{\theta+\delta}^2])}$$

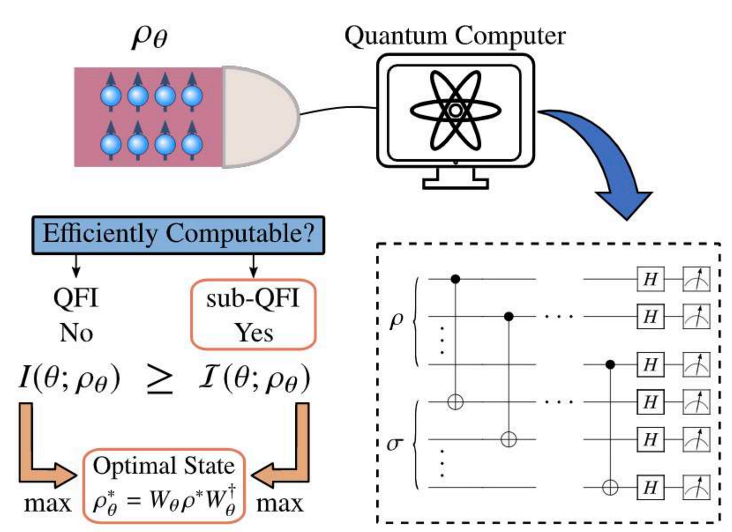
sub-QFI

$$\mathcal{I}(\theta;\rho_\theta) = 8\lim_{\delta\to 0}\frac{1-\sqrt{G(\rho_\theta,\rho_{\theta+\delta})}}{\delta^2}$$

computable with depth 2 quantum circuit and efficient classical post-processing

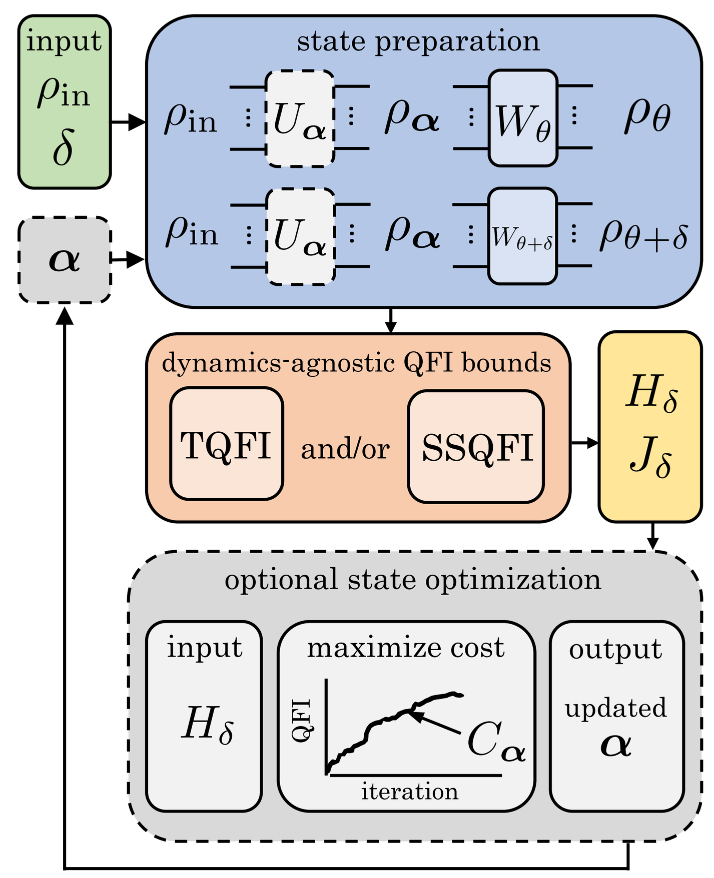
optimal state is known[2]

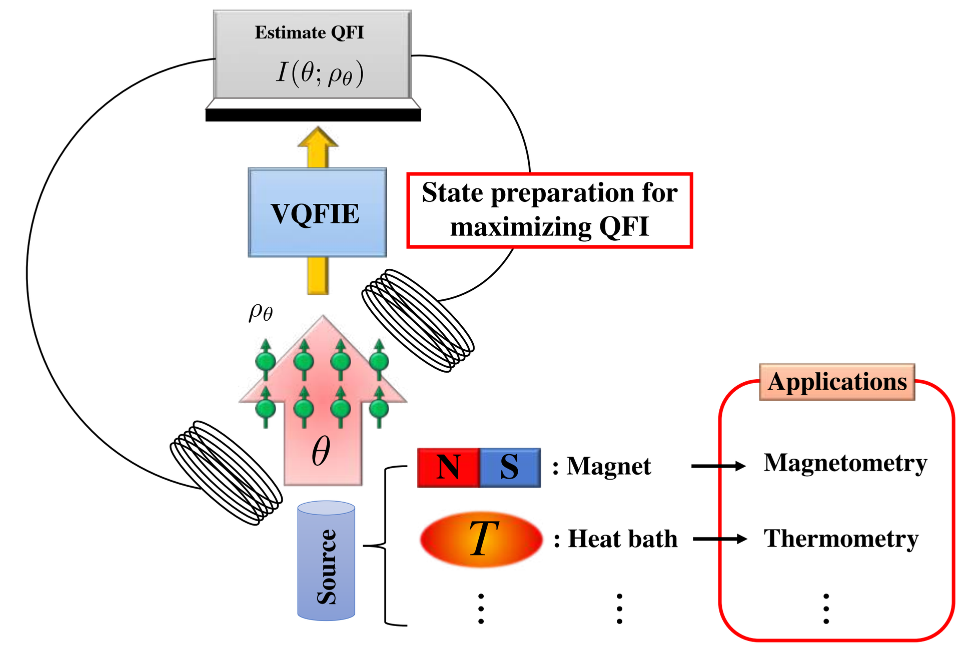




QFI and sub-QFI share global extrema

Variational QFI Estimation Algorithm

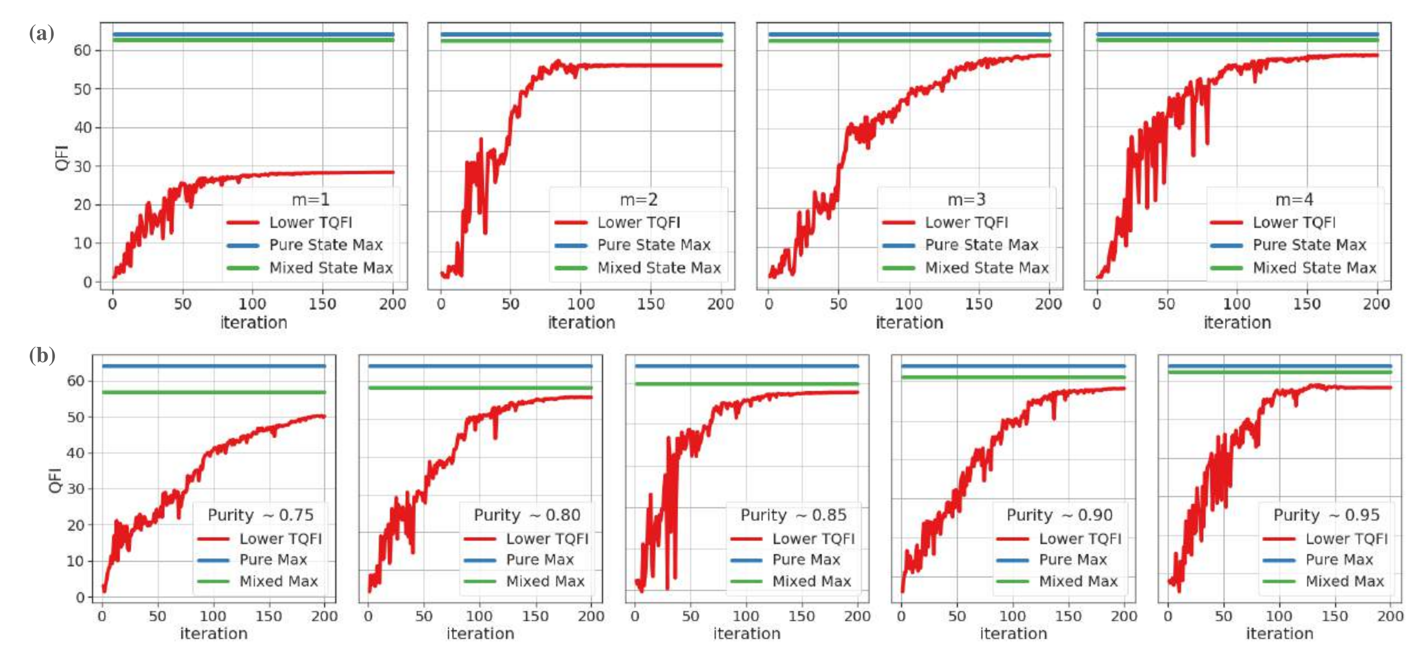




Left: overview of the VQFIE algorithm[3]. Note that one can use the algorithm to simply estimate the QFI. However, it can also be used to prepare the approximately optimal state for sensing

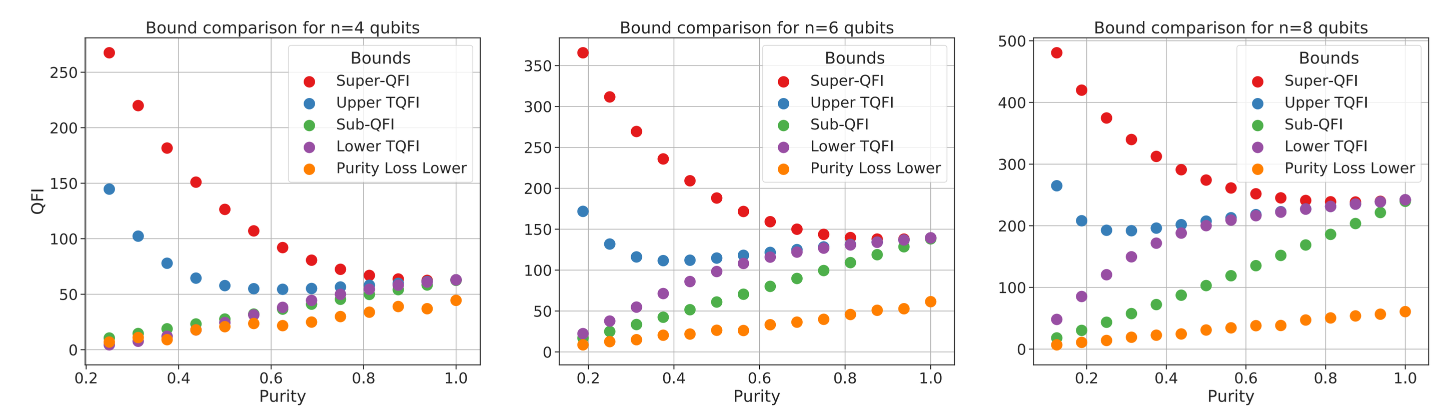
Right: schematic diagram of potential applications of VQFIE

Optimal State Preparation for Numerical Magnetometry Experiment



Top : we see the TQFI bounds tighten monotonically with the number of eigenvalues kept
Bottom : we see strong performance over a broad purity range, confirming VQFIE's utility

Bound Comparison



TQFI bounds were consistently tighter than sub- and super-QFI bounds as well as purity loss bounds

References

Please see our recent pre-prints related to this work for more details as well as a full bibliography.

TQFI
arXiv:2010.02904

sub-QFI
arXiv:2101.10144

VQFIE
arXiv:2010.10488