

# Continuous Variable Quantum Imaginary-Time Evolution Algorithm for Quantum Field Theories

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## Introduction

Simulating quantum field theories (QFTs) using quantum computers would be one of the most significant contributions of quantum computers since QFTs help us understand how Nature works. Although QFTs have been studied on quantum computers with discrete variables using algorithms such as variational quantum eigensolver (VQE) [1] the use of continuous variables (CVs) for studying the QFTs is a natural platform because the fields are also CVs. Quantum imaginary-time evolution (QITE) algorithm is used to find the ground state energy of a symmetry sector of a system that the converged state has non-zero overlap with initial state [2]. In this work, we study the use of QITE algorithm to calculate the ground and 1<sup>st</sup> excited state energies of a 1+1-dimensional interacting scalar QFT. We also explore methods and sources needed for implementing this algorithm on a photonic continuous-variable quantum hardware.

$$H = H_0 + H_{\text{int}} + H_{\text{c.t.}}$$

$$H_0 = \frac{1}{2} \int_0^L dx \left[ \pi^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + m^2 \phi^2 \right]$$

$$H_{\text{int}} = g \int_0^L dx \phi^4 \quad \text{Interacting}$$

$$H_{\text{c.t.}} = \frac{\delta_m}{2} \int_0^L dx \phi^2 \quad \text{Counter-term}$$

After discretization, for N=1 point toy model the scalar field becomes  $\phi(x) \rightarrow q$  and we use

$$H_0 = \frac{1}{2}(p^2 + q^2 - 1) = a^\dagger a$$

$$H = H_0 + gq^4$$

## CV QITE with Ancilla Qubit

Only Gaussian operators (up to second order of quadrature operators) can be represented on a CV quantum computer. The state for sth step of QITE algorithm

$$|\Psi[s]\rangle = \overbrace{e^{-\tau H} \dots e^{-\tau H}}^{s \text{ times}} |\Psi_0\rangle$$

$$|\Psi_s(\tau)\rangle \equiv \frac{e^{-\tau H} |\Psi[s-1]\rangle}{\|e^{-\tau H} |\Psi[s-1]\rangle\|}$$

$$\| |\Psi[s]\rangle - |\Psi_s(\tau)\rangle \|$$

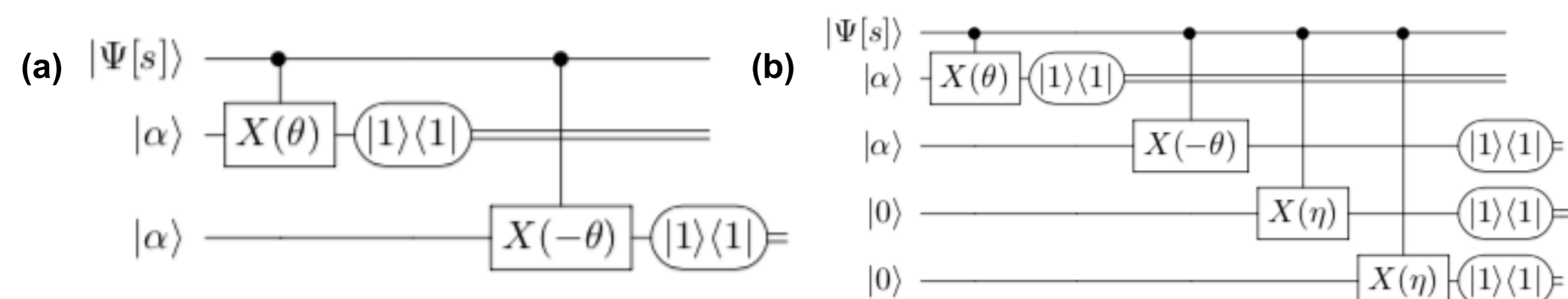
is minimized with

$$\gamma^2 = \frac{\langle q^2 H + H q^2 \rangle - 2\langle q^2 \rangle \langle H \rangle}{\langle q^4 \rangle - \langle q^2 \rangle^2} \tau$$

for small  $\tau$  and  $\gamma$ .

$$|\Psi[s]\rangle = \frac{(1 - \gamma^2 q^2 / 2) |\Psi[s-1]\rangle}{\|(1 - \gamma^2 q^2 / 2) |\Psi[s-1]\rangle\|}$$

Approximated with



The circuit (a) is used to calculate energy expectation value for the non-interacting Hamiltonian and circuit (b) is used for calculating energy expectation value of the interacting Hamiltonian and it requires addition of 2 more ancilla qubits. But recycling the qumodes after measurement reduces the number of required qumodes to only 4N qumodes for a general N point system.

$$P_{11} = \|\langle 1| \langle 1| e^{i\eta p_3 q} e^{i\eta p_4 q} |\Psi\rangle |0\rangle_3 |0\rangle_4 \|^2 \approx \frac{\eta^4}{4} \|q^2 |\Psi\rangle\|^2 = \frac{\eta^4}{4} \langle \Psi | q^4 | \Psi \rangle \longrightarrow g \langle \Psi | q^4 | \Psi \rangle = \frac{4g}{\eta^4} P_{11}$$

Other terms in  $\gamma^2$  can be calculated using similar strategy.

The energy expectation value of the interacting Hamiltonian is expressed in terms of probabilities of measurement outcomes projecting the ancillary modes onto photon number eigenstates for small  $\eta$  value.

## CV QITE with Beam Splitters

Adding beam splitters to the quantum circuit we can increase the probability of success to 37%, for  $N \sim 1/|\alpha^2|$  tries.

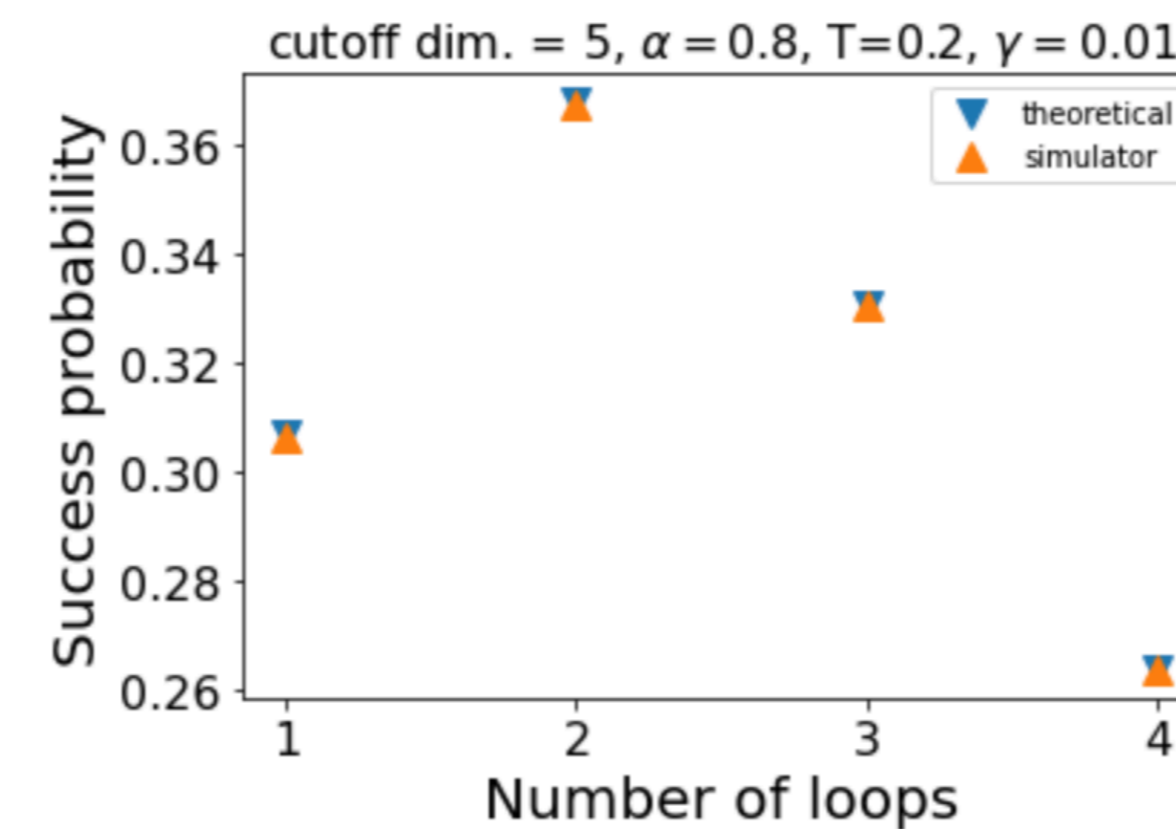
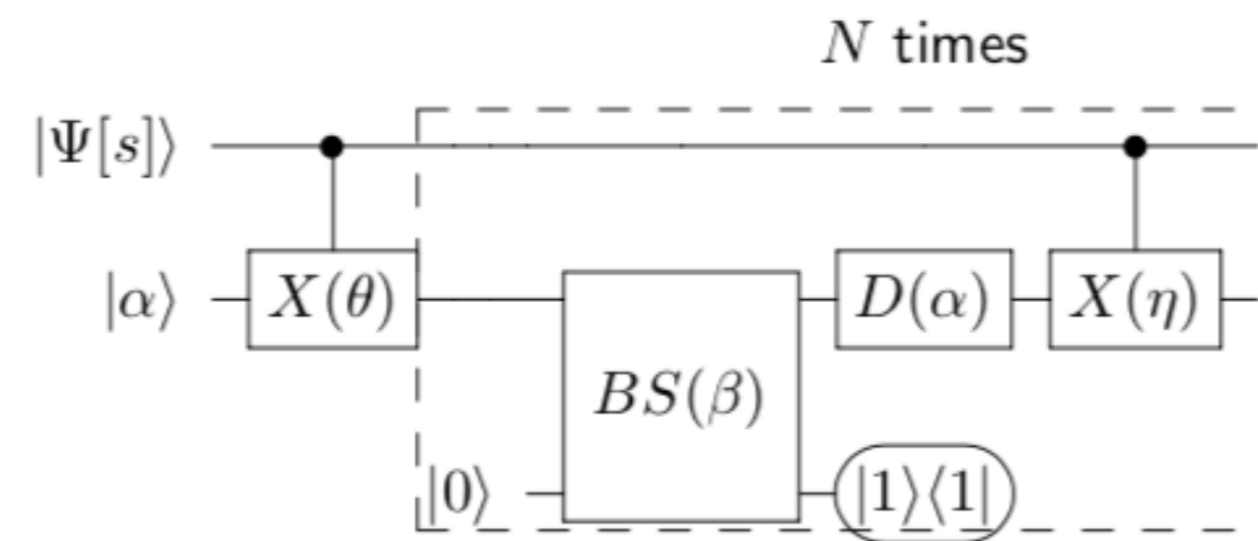


Figure shows success probability as a function of number of loops.

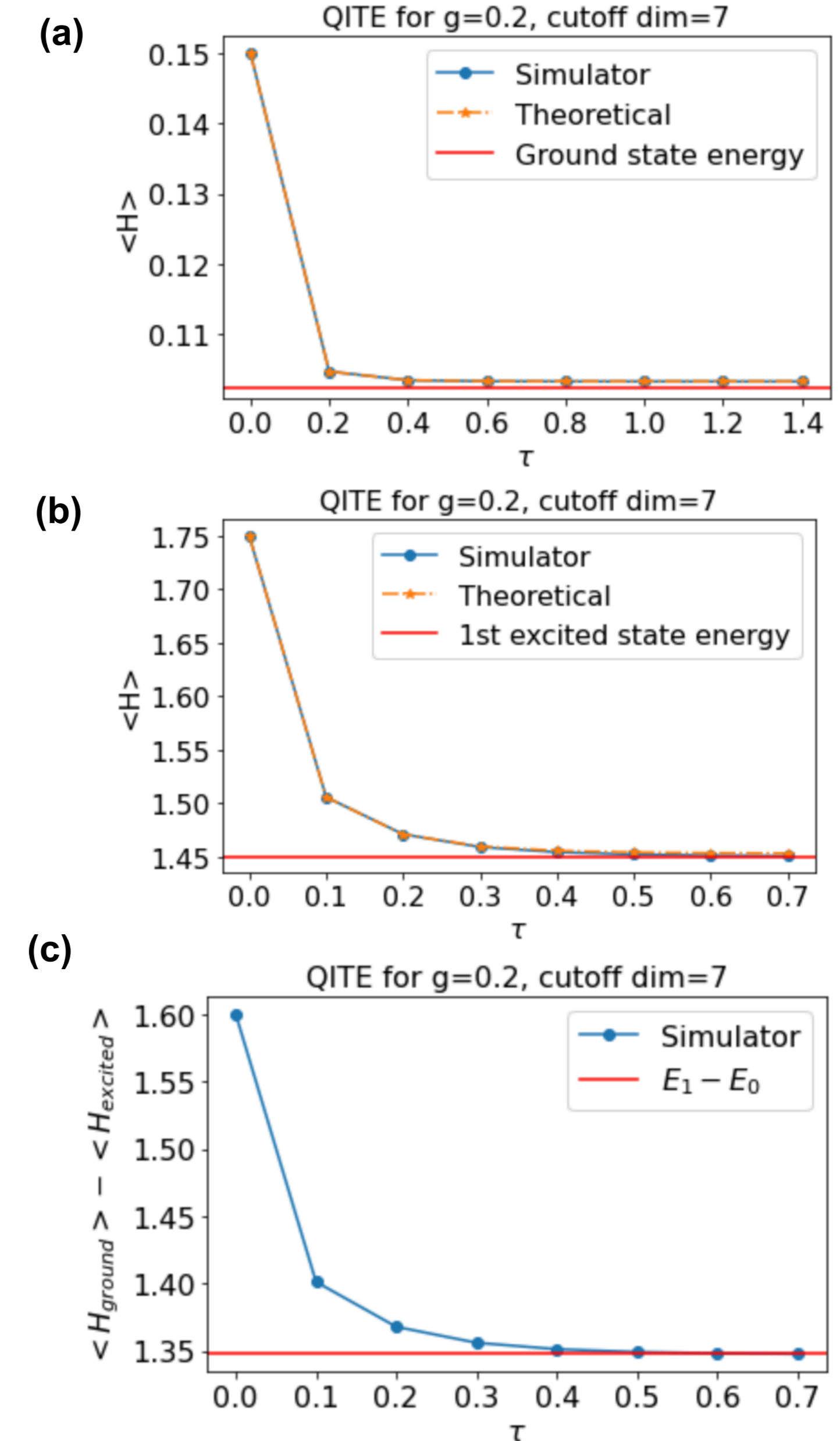
## Discussion

In this work, we explored methods to implement QITE algorithm on CV quantum computers to find the ground and first excited state energy of  $\phi^4$  interacting scalar field theory. We implemented the developed algorithms on Xanadu's photonic quantum computer simulator using Strawberryfields software for CV quantum computing and calculated the ground and first excited state energies of  $\phi^4$  interacting scalar field theory. As future work, we will implement this algorithm on Xanadu's photonic hardware, compare CV QITE algorithm with VQE using CVs and generalized this method for N point case.

## Bibliography

- [1] K. Yeter-Aydeniz, E. F. Dumitrescu, A. J. McCaskey, R. S. Bennink, R. C. Pooser, and G. Siopsis, "Scalar quantum field theories as a benchmark for near-term quantum computers", Phys. Rev. A **99**, 032306 (2019).  
 [2] M. Motta, C. Sun, A. T. K. Tan, M. J. O'Rourke, E. Ye, A. J. Minnich, F. G. S. L. Brandão, and G. K.-L. Chan, "Determining eigenstates and thermal states on a quantum computer using quantum imaginary time evolution", Nature Physics **16** (2020).

## Results



Energy vs. imaginary time for convergence to the ground ((a)) and 1<sup>st</sup> excited state ((b)) energies and the difference between ground and 1<sup>st</sup> excited state energy (the mass gap) ((c)) obtained using QITE algorithm.

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